Publications and preprints

Nicolas Ginoux - May 3, 2019

- **Articles:**

[23] Bernd Ammann and Nicolas Ginoux, *Some examples of Dirac-harmonic maps*, Lett. Math. Phys. **109** (2019), no. 5, 1205–1218

[22] Nicolas Ginoux, Georges Habib and Ines Kath, *A splitting theorem for Riemannian manifolds of generalised Ricci-Hessian type*, submitted

[21] Matthias Becker, Nicolas Ginoux, Sébastien Martin and Zsuzsanna Róka, *Tire Noise Optimization Problem: a Mixed Integer Linear Program Approach*, submitted

[20] Nicolas Ginoux and Olaf Müller, *Global solvability of massless Dirac-Maxwell systems*, Ann. Inst. H. Poincaré Anal. Non Linéaire **35** (2018), no. 6, 1645–1654

[19] Fida El Chami, Nicolas Ginoux, Georges Habib and Roger Nakad, *Rigidity results for Riemannian spin^c manifolds with foliated boundary*, Results in Math. **72** (2017), no. 4, 1773–1806

[18] Fida El Chami, Nicolas Ginoux, Georges Habib and Roger Nakad, *Rigidity results for spin manifolds with foliated boundary*, J. Geom. **107** (2016), no. 3, 533–555

[17] Nicolas Ginoux, *About the Lorentzian Yamabe problem*, Geom. Dedicata **174** (2015), 287–309

[16] Nicolas Ginoux, Georges Habib and Simon Raulot, *A new upper bound for the Dirac operator on hypersurfaces*, Pacific J. Math. **278** (2015), no. 1, 79–101

[15] Bernd Ammann and Nicolas Ginoux, *Dirac-harmonic maps from index theory*, Calc. Var. Part. Diff. Eq. **47** (2013), no. 3-4, 739–762

[14] Christian Bär and Nicolas Ginoux, *Classical and quantum fields on Lorentzian manifolds*, in: C. Bär et al. (eds): "Global Differential Geometry", Springer Proceedings in Mathematics **17** (2012), no. 2, 359–400

[13] Nicolas Ginoux and Georges Habib, *The spectrum of the twisted Dirac operator on Kähler submanifolds of the complex projective space*, manuscripta math. **137** (2012), no. 1-2, 215–231

[12] Nicolas Ginoux and Uwe Semmelmann, *Imaginary Kählerian Killing spinors I*, Ann. Glob. Anal. Geom. **40** (2011), no. 4, 467–495

[11] Nicolas Ginoux and Jean-François Grosjean, *Almost harmonic spinors*, C. R. Math. Acad. Sci. Paris **348** (2010), no. 13-14, 811–814

[10] Nicolas Ginoux and Georges Habib, *A spectral estimate for the Dirac operator on Riemannian flows*, Cent. Eur. J. Math. **8** (2010), no. 5, 950–965

[9] Nicolas Ginoux and Georges Habib, *Remarques sur les spineurs de Killing transversaux*, C. R. Math. Acad. Sci. Paris **346** (2008), no. 11-12, 657–659

[8] Nicolas Ginoux and Georges Habib, *Geometric aspects of transversal Killing spinors on Riemannian flows*, Abh. Math. Sem. Univ. Hamburg **78** (2008), 69–90

[7] Nicolas Ginoux, *The spectrum of the Dirac operator on* $\mathrm{SU}_2/\mathrm{Q}_8$, manuscripta math. **125** (2008), no. 3, 383–409

[6] Florin Belgun, Nicolas Ginoux and Hans-Bert Rademacher, *A singularity theorem for twistor-spinors*, Ann. Inst. Fourier **57** (2007), no. 4, 1135–1159

[5] Nicolas Ginoux, *Dirac operators on Lagrangian submanifolds*, J. Geom. Phys. **52** (2004), no. 4, 480–498

[4] Nicolas Ginoux, *Remarques sur le spectre de l'opérateur de Dirac*, C. R. Acad. Sci. Paris Sér. I **337** (2003), no. 1, 53–56

[3] Nicolas Ginoux, *Une nouvelle estimation extrinsèque du spectre de l'opérateur de Dirac*, C. R. Acad. Sci. Paris Sér. I **336** (2003), no. 10, 829–832

[2] Nicolas Ginoux, *Reilly-type spinorial inequalities*, Math. Z. **241** (2002), no. 3, 513–525

[1] Nicolas Ginoux and Bertrand Morel, *On eigenvalue estimates for the submanifold Dirac operator*, Int. J. Math. **13** (2002), no. 5, 533–548

• **Books:**

[B2] Nicolas Ginoux, *The Dirac spectrum*, Lecture Notes in Mathematics **1976** (2009), Springer

[B1] Christian Bär, Nicolas Ginoux and Frank Pfäffle, *Wave equations on Lorentzian manifolds and quantization*, ESI Lectures in Mathematics and Physics (2007), EMS Publishing House

• **Proceedings:**

[P4] Matthias Becker, Nicolas Ginoux, Sébastien Martin and Zsuzsanna Róka, *Optimization of Tire Noise by Solving an Integer Linear Program (ILP)*, in: 2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC 2016), October 9-12, 2016, Budapest

[P3] Christian Bär and Nicolas Ginoux, CCR- *versus* CAR-*quantization on curved spacetimes*, in: F. Finster et al. (eds.): "Quantum Field Theory and Gravity", Birkhäuser, 183–206, 2012

[P2] Nicolas Ginoux, *Linear wave equations*, in: C. Bär et K. Fredenhagen (eds.): "Quantum field theory on curved spacetimes", Lecture Notes in Physics **786** (2009), 59–84, Springer

[P1] Nicolas Ginoux, *Reilly-type spinorial inequalities*, in: J.-P. Bourguignon et al. (eds.): "Dirac operators: Yesterday and Today", 263–269, International Press, 2005

• **Theses:**

[H] Nicolas Ginoux, *Analysis on Kähler and Lorentzian manifolds*, Habilitation thesis, Universität Regensburg, 2014

[D] Nicolas Ginoux, *Opérateurs de Dirac sur les sous-variétés*, PhD thesis, Université Henri Poincaré - Nancy 1, 2002

[M] Nicolas Ginoux, *Géométrie hermitienne et géométrie spinorielle conforme*, Master thesis, Université Henri Poincaré - Nancy 1, 1998

Comments on publications

23. **Some examples of Dirac-harmonic maps** [23]

   • Bernd Ammann and Nicolas Ginoux, *Some examples of Dirac-harmonic maps*, Lett. Math. Phys. **109** (2019), no. 5, 1205–1218

   Abstract: We discuss a method to construct Dirac-harmonic maps developed by Jürgen Jost, Xiaohuan Mo and Miaomiao Zhu in *Some explicit constructions of Dirac-harmonic maps*, J. Geom. Phys. **59** (2009), no. 11, 1512–1527. The method uses harmonic spinors and twistor spinors, and mainly applies to Dirac-harmonic maps of codimension 1 with target spaces of constant sectional curvature. Before the present article, it remained unclear when the conditions of the theorems in *Some explicit constructions of Dirac-harmonic maps*, J. Geom. Phys. **59** (2009), no. 11, 1512–1527, were fulfilled. We show that for isometric immersions into spaceforms, these conditions are fulfilled only under special assumptions. In several cases we show the existence of solutions.

   Comments: Despite the high complexity of the Dirac-harmonic-map-equations, it is possible to produce nontrivial examples via an elementary ansatz for the spinor component involving harmonic and twistor spinors; this was the main object of the article *Some explicit constructions of Dirac-harmonic maps* by Jürgen Jost, Xiaohuan Mo and Miaomiao Zhu. However, the existence of a nonzero twistor spinor imposes restrictions on the geometry of the underlying Riemannian manifold, so that it was not completely obvious from their article how large the category of examples produced that way was. We show in our article that actually only few examples can be constructed with that method, and that more or less only one example of a coupled Dirac-harmonic map (i.e. with nonharmonic mapping component) up to isometry comes out.

22. **A splitting theorem for Riemannian manifolds of generalised Ricci-Hessian type** [22]

   • Nicolas Ginoux, Georges Habib and Ines Kath, *A splitting theorem for Riemannian manifolds of generalised Ricci-Hessian type*, submitted

   Abstract: In this paper, we study and partially classify those Riemannian manifolds carrying a non-identically vanishing function f whose Hessian is minus f times the Ricci-tensor of the manifold.

   Comments: While struggling with technical and lengthy computations in our common project about so-called *skew Killing spinors*, Georges Habib, Ines Kath and I stumbled on a particularly simple-to-write equation satisfied by some function f on the background manifold M, namely $\nabla^2 f = -f \cdot \mathrm{Ric}$, where $\nabla^2 f$ and Ric are the Hessian and Ricci tensors respectively. Surprisingly enough, we discovered that this "Obata-like" equation had never been considered before, though exhibiting similarities with e.g. almost Ricci soliton warped products. Motivated by a better understanding of the geometric background behind skew-Killing spinors, we attempted to classify in some way manifolds with a nontrivial solution to that equation. It turns out that the naked equation does not tell much about the background geometry but that a whole category of examples, namely Riemannian products of 2-dimensional spaceforms with Ricci-flat manifolds, can be recovered provided sufficiently many symmetries are assumed to exist.

21. **Tire Noise Optimization Problem: a Mixed Integer Linear Program Approach** [21]

   • Matthias Becker, Nicolas Ginoux, Sébastien Martin and Zsuzsanna Róka, *Tire Noise Optimization Problem: a Mixed Integer Linear Program Approach*, submitted

   Abstract: We present a Mixed Integer Linear Program (MILP) approach in order to model the non linear problem of minimizing the tire noise. We first take more industrial constraints into account than in a former work of the authors. Then we associate a Branch-and-Cut algorithm to the MILP to obtain exact solutions. We compare our experimental results with those obtained by other methods.

   Comments: We develop and refine the model introduced in [P4] for the minimization of the noise produced by a tire. In particular, we are able to handle a larger number of constraints and, above all, to compute the real noise in contrast to [P4] where only an approximated noise was optimized.

20. **Global solvability of massless Dirac-Maxwell systems** [20]

   • Nicolas Ginoux and Olaf Müller, *Global solvability of massless Dirac-Maxwell systems*, Ann. Inst. H. Poincaré Anal. Non Linéaire **35** (2018), no. 6, 1645–1654

   Abstract: We consider the Cauchy problem of massless Dirac-Maxwell equations on an asymptotically flat background and give a global existence and uniqueness theorem for initial values small in an appropriate weighted Sobolev space.

Comments: The Dirac-Maxwell equations describe the interaction between an electron (modelled by a spinor field) and an electromagnetic field. They form a system of coupled non-linear partial differential equations that are gauge-invariant and, in case the spacetime dimension is 4 and the mass vanishes, are also conformally invariant. Using both facts and the *Penrose-embedding* of Minkowski spacetime, Choquet-Bruhat and Christodoulou [1981] reduced global existence for solutions to a system of massless Dirac-Maxwell equations coupled with a Higgs field on the 4-dimensional Minkowski spacetime to a *local* existence result on a compact domain of the Einstein cylinder, the latter following in a straightforward way from the local theory for symmetric hyperbolic systems. Here we adapt their approach to a more general category of spacetimes – namely *asymptotically flat* spacetimes – and for massless Dirac-Maxwell systems involving more than one electron. The main difficulty in that case is that there is in general no longer any *smooth* conformal and precompact embedding of the spacetime into a Cauchy-compact product spacetime since a singularity occurs along a given spacelike Cauchy hypersurface. We avoid this by restricting ourselves on the chronological future or past of a given Cauchy hypersurface and show that, providing the total charge of the electrons vanishes and some constraint equation is satisfied along a Cauchy hypersurface *contained in the future or the past* of the original hypersurface, the massless Dirac-Maxwell system has a global solution as soon as the initial condition is sufficiently small in some weighted Sobolev norm.

19. **Rigidity results for Riemannian spin^c manifolds with foliated boundary** [19]

   • Fida El Chami, Nicolas Ginoux, Georges Habib and Roger Nakad, *Rigidity results for Riemannian spin^c manifolds with foliated boundary*, Results in Math. **72** (2017), no. 4, 1773–1806

   Abstract: Given a Riemannian spin^c manifold whose boundary is endowed with a Riemannian flow, we show that any solution of the basic Dirac equation satisfies an integral inequality depending on geometric quantities, such as the mean curvature and the O'Neill tensor. We then characterize the equality case of the inequality when the ambient manifold is a domain of a Kähler-Einstein manifold or a Riemannian product of a Kähler-Einstein manifold with $\mathbb{R}$ (or with the circle $\mathbb{S}^1$).

   Comments: This paper extends [18] to the more general situation where only spin^c (instead of spin) structures are assumed to exist on the manifolds under consideration. We originally aimed at characterizing boundaries of geodesic balls in complex spaceforms in terms of curvature conditions. We faced several difficulties, the first of which consisting in the choice of auxiliary line bundle and connection for the different spin^c structures on the boundary of a given spin^c manifold. It turns out that there is in our context a natural setting for inducing spin^c structures and connections and that the only freedom left can be expressed in terms of a real one-form on the boundary whose exterior differential is related to the curvature of some line bundle. Setting up this natural framework, it turns out that no more than two geometric situations are of interest for our study, the case where the manifold with boundary is Kähler and the case where it lies in the product of a Kähler manifold with a line or a circle. The rigidity results we obtain then follow in a much less technical way because of the much richer structure we have at hand.

18. **Rigidity results for spin manifolds with foliated boundary** [18]

   • Fida El Chami, Nicolas Ginoux, Georges Habib and Roger Nakad, *Rigidity results for spin manifolds with foliated boundary*, J. Geom. **107** (2016), no. 3, 533–555

   Abstract: In this paper, we consider a compact Riemannian manifold whose boundary is endowed with a Riemannian flow. Under a suitable curvature assumption depending on the O'Neill tensor of the flow, we prove that any solution of the basic Dirac equation is the restriction of a parallel spinor field defined on the whole manifold. As a consequence, we show that the flow is a local product. In particular, in the case where solutions of the basic Dirac equation are given by basic Killing spinors, we characterize the geometry of the manifold and the flow.

   Comments: Inspired by Hijazi-Montiel and Raulot's fundamental geometric characterisations of the round sphere as the boundary of a compact manifold satisfying curvature assumptions, we aimed at adapting their methods to the more technical situation where the boundary is *foliated*. Lots of different model geometries are at hand in that case and we concentrated on the simplest model where the boundary is the Riemannian product of an $\mathbb{S}^1$ with a closed manifold. Although the techniques are more involved than in the non-foliated case – since we have to e.g. consider *basic* objects, in particular the basic Dirac operator – we obtain "spinor-free" characterisations of such product boundaries under assumptions depending only on intrinsic and extrinsic curvature quantities such as the scalar and mean curvatures as well as O'Neill tensor of the foliation.

17. **About the Lorentzian Yamabe problem** [17]

   • Nicolas Ginoux, *About the Lorentzian Yamabe problem*, Geom. Dedicata **174** (2015), 287–309

   Abstract: We investigate the solutions to the Yamabe problem on globally hyperbolic spacetimes. On standard static spacetimes, we prove the existence of global solutions and show with the help of examples that uniqueness does not hold in general.

   Comments: Though intensively studied for Riemannian metrics, the Yamabe problem, asking for the existence of constant-scalar-curvature-metrics in a given conformal class, had not been considered for other signatures. Here we restrict ourselves to globally hyperbolic (Lorentzian) manifolds, where the Yamabe equation is hyperbolic and hence can be hoped to be solved globally. As a preliminary study, we look at spacetimes conformally equivalent to standard static (that is, product) ones with compact spacelike slice(s), where the Yamabe problem can be completely solved in terms of the *Yamabe invariant* of the slice. The main difficulty when investigating arbitrary globally hyperbolic spacetimes consists in *controlling the sign* of the solution for long times, problem that does not appear in the Riemannian context because of the maximum principle.

16. **A new upper bound for the Dirac operator on hypersurfaces** [16]

   • Nicolas Ginoux, Georges Habib and Simon Raulot, *A new upper bound for the Dirac operator on hypersurfaces*, Pacific J. Math. **278** (2015), no. 1, 79–101

   Abstract: We prove a new upper bound for the first eigenvalue of the Dirac operator of a compact hypersurface in any Riemannian spin manifold carrying a non-trivial twistor spinor without zeros on the hypersurface. The upper bound is expressed as the first eigenvalue of a drifting Schrödinger operator on the hypersurface. Moreover, using a recent approach developed by O. Hijazi and S. Montiel, we completely characterize the equality case when the ambient manifold is the standard hyperbolic space.

   Comments: As for the scalar Laplacian, the smallest eigenvalues of the Dirac operator on a compact hypersurface in a spaceform can be bounded in terms of the mean curvature of the immersion (see in particular [2, 3]). But, unlike the scalar Laplacian, no *conformal* upper bound can hold for the smallest non-zero Dirac-eigenvalue [Ammann & Jammes '05]. Still the question remained open whether an upper bound could be given for all hypersurfaces of manifolds conformally embedded in the round sphere of the same dimension (in particular all three spaceforms), as is the case for the Laplacian [El Soufi & Ilias '92]. In this article, we proved that, under even *weaker assumptions* on the ambient manifold, there exists an upper bound for the smallest Dirac-eigenvalue that *enhances all above-mentioned ones*. This upper bound coincides with the smallest eigenvalue of an *a priori* unrelated scalar operator of the form drifting Laplacian plus (geometrically defined) potential. Moreover – and as was not obvious from Hijazi and Montiel's analogous result in Euclidean space – we could show that *only round hyperspheres satisfy the equality* in the estimate when the ambient manifold is the *hyperbolic space*.

15. **Dirac-harmonic maps from index theory** [15]

   • Bernd Ammann and Nicolas Ginoux, *Dirac-harmonic maps from index theory*, Calc. Var. Part. Diff. Eq. **47** (2013), no. 3-4, 739–762

   Abstract: We prove existence results for Dirac-harmonic maps using index theoretical tools. They are mainly interesting if the source manifold has dimension 1 or 2 modulo 8. Our solutions are uncoupled in the sense that the underlying map between the source and target manifolds is a harmonic map.

   Comments: Although Dirac-harmonic maps, which can be thought of as the fermionic analogue of harmonic maps, were introduced in the mathematical literature a long time ago, only few isolated examples were known before our work. This is due to the non-trivial way a Dirac-harmonic map couples a map between manifolds and a (twisted) spinor field. For the first time, we could show that, starting from any *harmonic* map f_0 for which the *index* of an associated Dirac operator *does not vanish*, the Atiyah-Singer index theorem yields automatically the *existence of a nontrivial Dirac-harmonic map* attached to f_0. That result allows to *produce whole families of Dirac-harmonic maps* that contain virtually all previously known examples.

14. **Classical and quantum fields on Lorentzian manifolds** [14] (shorter version in [P3])

   • Christian Bär and Nicolas Ginoux, *Classical and quantum fields on Lorentzian manifolds*, in: C. Bär et al. (eds): "Global Differential Geometry", Springer Proceedings in Mathematics **17** (2012), no. 2, 359–400.

Abstract: We construct bosonic and fermionic locally covariant quantum field theories on curved backgrounds for large classes of fields. We investigate the quantum field and n-point functions induced by suitable states.

Comments: Loosely speaking, locally covariant quantum field theory (QFT) associates algebras to solutions of some linear PDE on the open subsets of a spacetime. First formulated in the unified framework of categories and functors by R. Brunetti, K. Fredenhagen and R. Verch, it had been developed for various operators and for various purposes, however no attempt had been made to perform it for the "most general" linear operators. This is what we tackled, showing a.o. that *the existence of so-called Green's operators was the only necessary ingredient* for constructing so-called bosonic locally covariant QFT's.

13. **The spectrum of the twisted Dirac operator on Kähler submanifolds of the complex projective space** [13]

 • Nicolas Ginoux and Georges Habib, *The spectrum of the twisted Dirac operator on Kähler submanifolds of the complex projective space*, manuscripta math. **137** (2012), no. 1-2, 215–231

 Abstract: We establish an upper estimate for the small eigenvalues of the twisted Dirac operator on Kähler submanifolds in Kähler manifolds carrying Kählerian Killing spinors. We then compute the spectrum of the twisted Dirac operator of the canonical embedding $\mathbb{C}\mathrm{P}^d \to \mathbb{C}\mathrm{P}^n$ in order to test the sharpness of the upper bounds.

 Comments: This work establishes a parallel with [2, 3] but for Kähler submanifolds of complex spaceforms. Namely, given a compact Kähler spin submanifold M of e.g. $\mathbb{C}\mathrm{P}^n$, what kind of relationship is there between the small eigenvalues of some Dirac operator of M and geometric data derived from the immersion (second fundamental form)? In this article, we looked more specifically at a particular twisted Dirac operator that appears naturally when restricting spinors from $\mathbb{C}\mathrm{P}^n$ to M. As in the spirit of [1], forgetting first about the extrinsic geometry, we determined an *optimal intrinsic lower bound* for the smallest eigenvalues of that twisted Dirac operator in terms of *curvature* quantities. Coming back to our original question, we produced an extrinsic upper eigenvalue bound (for those smallest eigenvalues) that, surprisingly enough, *only depends on the dimension of* M (and not on the second fundamental form, as one would expect). To see how sharp our upper bound was, we computed the spectrum of the twisted Dirac operator on the simple example at hand, namely a totally geodesically embedded $M = \mathbb{C}\mathrm{P}^d$ in $\mathbb{C}\mathrm{P}^n$ and showed that, due to the presence of curvature terms, the estimate can be *sharp only in low codimensions.*

12. **Imaginary Kählerian Killing spinors I** [12]

 • Nicolas Ginoux and Uwe Semmelmann, *Imaginary Kählerian Killing spinors I*, Ann. Glob. Anal. Geom. **40** (2011), no. 4, 467–495

 Abstract: We describe and to some extent characterize a new family of Kähler spin manifolds admitting non-trivial imaginary Kählerian Killing spinors.

 Comments: The smallest eigenvalue of the Dirac operator of a compact Kähler spin manifold can be estimated from below in terms of the scalar curvature of the underlying space [K.-D. Kirchberg '86]. In case the smallest eigenvalue coincides with the lower bound (the estimate is then said to be *sharp*), particular spinor fields exist on the manifolds, called *Kählerian Killing spinors.* Kählerian Killing spinors come with a constant, which is either real or purely imaginary. For the first time, we were able to partially *classify* those complete Kähler spin manifolds carrying Kählerian Killing spinors with imaginary associated constant and to describe whole *families of examples in each (odd) complex dimension*, revealing unsuspected connections with transversal parallel spinors (see [8]).

11. **Almost harmonic spinors** [11]

 • Nicolas Ginoux and Jean-François Grosjean, *Almost harmonic spinors*, C. R. Math. Acad. Sci. Paris **348** (2010), no. 13-14, 811–814

 Abstract: We show that any closed spin manifold not diffeomorphic to the two-sphere admits a sequence of volume-one-Riemannian metrics for which the smallest non-zero Dirac eigenvalue tends to zero. As an application, we compare the Dirac spectrum with the conformal volume.

 Comments: Introduced by Peter Li and Shing-Tung Yau in the 80s, the conformal volume of a compact Riemannian manifold is a conformal invariant computed out of the volume of the metrics induced from conformal immersions in spheres of sufficiently high dimensions. It has been shown to provide an upper bound for the smallest positive Laplace eigenvalue of unit-volume metrics. For the Dirac operator, only conformal lower eigenvalue bounds can be expected and it had been a long-standing open question to determine whether the conformal volume can be such a lower bound or not. We showed that, apart from the case where the manifold is the 2-dimensional sphere, the answer to the question is always *negative.* This statement comes as an easy corollary of a much more general result, namely that, the 2-sphere left aside, 0 *can be made as close* to the first positive Dirac eigenvalue as one wants by choosing the unit-volume-metric appropriately in the conformal class.

10. **A spectral estimate for the Dirac operator on Riemannian flows** [10]

    • Nicolas Ginoux and Georges Habib, *A spectral estimate for the Dirac operator on Riemannian flows*, Cent. Eur. J. Math. **8** (2010), no. 5, 950–965

    Abstract: We give a new upper bound for the smallest eigenvalues of the Dirac operator on a Riemannian flow carrying transversal Killing spinors. We also derive an estimate on Sasakian and on 3-dimensional manifolds and partially classify those satisfying the limiting case. Finally, we compare our estimate with a lower bound in terms of a natural tensor depending on the eigenspinor.

    Comments: This work can be seen as the "dual" version of [2, 3]: here we consider compact Riemannian manifolds that *submerge* over spaceforms and investigate the relationship between their Dirac spectrum and the O'Neill tensor of the submersion. The basic remark is that such submerged manifolds carry so-called *transversal Killing spinors* which lift the Killing spinors living on the spaceform downstairs. More generally we start with so-called Riemannian foliations, which are locally Riemannian submersions with one-dimensional fibres, and under the assumption of existence of transversal Killing spinors (defined in [8]) prove a very *general upper bound* for the smallest Dirac eigenvalue depending on the *transverse structure* of the flow. In particular cases, namely when the manifold is Sasaki or 3-dimensional, we simplify the estimates and show on examples that they are *optimal.* In 3 dimensions, we can even – with the help of [8] – *classify* those limiting manifolds. As an unexpected by-product, we obtain the *first known example* so far of a Riemannian spin manifold where the standard Friedrich inequality is *not* sharp but where a finer estimate by Oussama Hijazi and in terms of the so-called *energy-momentum* tensor is.

9. **Remarques sur les spineurs de Killing transversaux** [9]

    • Nicolas Ginoux and Georges Habib, *Remarques sur les spineurs de Killing transversaux*, C. R. Math. Acad. Sci. Paris **346** (2008), no. 11-12, 657–659

    Abstract: We show necessary conditions for the existence of transversal Killing spinors on a spin manifold endowed with a Riemannian flow.

    Comments: In the continuation of [8], we put to light finer integrability conditions for transversal Killing spinors, in particular in dimension 3 where Yves Carrière's classification of Riemannian flows can be used.

8. **Geometric aspects of transversal Killing spinors on Riemannian flows** [8]

    • Nicolas Ginoux and Georges Habib, *Geometric aspects of transversal Killing spinors on Riemannian flows*, Abh. Math. Sem. Univ. Hamburg **78** (2008), 69–90

    Abstract: We study a Killing spinor type equation on spin Riemannian flows. We prove integrability conditions and partially classify those Riemannian flows M carrying non-trivial solutions to that equation in case M is a local Riemannian product, a Sasakian manifold or 3-dimensional.

    Comments: This article is concerned with the geometric properties related to so-called transversal Killing spinors, that generalize what can be obtained by lifting (locally) Killing spinors when a given manifold submerges over a spaceform. We start by considering a general Killing-spinor-type equation on Riemannian flows, that are one-dimensional foliations which locally can be seen as Riemannian submersions. After deriving rather involved *curvature properties* linked to transversal Killing spinors on arbitrary Riemannian flows, we look at particular cases such as local Riemannian products, Sasaki or 3-dimensional flows. Surprisingly enough, there are examples of such transversal Killing spinors that are *not* lifts of Killing spinors on the base, even locally – for instance Berger spheres, where the transversal Killing-spinor-equation relates with the Killing-spinor-one. In some situations, we can *classify* – up to "standard" deformations of the flow – those flows admitting non-zero transversal Killing spinors, obtaining a relatively vast landscape. Parts of the classification were completed in [9].

7. **The spectrum of the Dirac operator on** $\mathrm{SU}_2/\mathrm{Q}_8$ [7]

• Nicolas Ginoux, *The spectrum of the Dirac operator on* $\mathrm{SU}_2/\mathrm{Q}_8$, manuscripta math. **125** (2008), no. 3, 383–409

Abstract: We compute the fundamental Dirac operator for the three-parameter-family of homogeneous Riemannian metrics and the four different spin structures on $\mathrm{SU}_2/\mathrm{Q}_8$, where Q_8 denotes the group of quaternions. We deduce its spectrum for the Berger metrics and show the sharpness of Christian Bär's upper bound for the smallest Dirac eigenvalue in the particular case where $\mathrm{SU}_2/\mathrm{Q}_8$ is a homogeneous minimal hypersurface of S^4.

Comments: This work can be seen as the natural continuation of [4], where hypersurfaces with two constant principal curvatures in the round sphere had been considered. Here we look at the quotient $\mathrm{SU}_2/\mathrm{Q}_8$, which is the simplest hypersurface with three constant principal curvatures in the round sphere, and investigate the sharpness of Christian Bär's upper bound for the smallest Dirac eigenvalue in terms of the L^2-norm of the mean curvature. Using representation-theoretical techniques, we *compute the Dirac spectrum* of a 1-parameter-family of Riemannian metrics (called *Berger metrics*), finitely many of which come from an isometric embedding in S^4 which is then minimal and for which Christian Bär's upper bound is *attained*. It should be mentioned that there is still a 1-parameter family of Riemannian metrics, standing "transversely" to the Berger one, for which the Dirac spectrum cannot be computed explicitly and for which we cannot (for the moment) prove the sharpness of Christian Bär's upper bound.

6. **A singularity theorem for twistor-spinors** [6]

   • Florin Belgun, Nicolas Ginoux and Hans-Bert Rademacher, *A singularity theorem for twistor-spinors*, Annales de l'Institut Fourier **57** (2007), no. 4, 1135–1159

   Abstract: We study compact spin orbifolds with finite singularity set carrying twistor-spinors. We show that any non-trivial twistor-spinor admits at most one zero which is singular unless the orbifold is conformally equivalent to a round sphere. We show the sharpness of our results through examples.

   Comments: Following earlier work by Wolfgang Kühnel and Hans-Bert Rademacher, the search for non-conformally flat manifolds admitting non-zero twistor-spinors had brought partial but not completely satisfying results. At the same time, technical considerations on zeros of twistor-spinors had for instance left the issue open whether twistor-spinors on the Eguchi-Hanson space can be "extended" onto some bigger space or not. In our paper, we brought a positive answer to that question by treating twistor-spinors in the general context of orbifolds and showing a very nice *geometric inequality* between the *orders of the singularities* linked with the zeros of twistor-spinors, recovering on the way results by André Lichnerowicz on manifolds. Moreover, as far as we know, we gave in this paper the first *detailed description of spin structures on orbifolds* and the first characterization of their existence.

5. **Dirac operators on Lagrangian submanifolds** [5]

   • Nicolas Ginoux, *Dirac operators on Lagrangian submanifolds*, J. Geom. Phys. **52** (2004), no. 4, 480–498

   Abstract: We study a natural Dirac operator on a Lagrangian submanifold of a Kähler manifold. We first show that its square coincides with the Hodge - de Rham Laplacian provided the complex structure identifies the spin structures of the tangent and normal bundles of the submanifold. We then give extrinsic estimates for the eigenvalues of that operator and discuss some examples.

   Comments: A Lagrangian submanifold M in a Kähler manifold $\widetilde{M}$ is an immersed (real) submanifold whose normal bundle is identified with its tangent bundle via the ambient complex structure. We first show that, assuming both M and $\widetilde{M}$ to be spin, the Dirac operator of M twisted with the spinor bundle of the normal bundle identifies with the *Euler operator* $d+\delta$ of M as soon as the isomorphism $TM \to T^{\perp}M$ given by the complex structure lifts to the spin level. In the case where $\widetilde{M} = \mathbb{C}\mathrm{P}^n$ and M is compact, we prove in the spirit of [C. Bär '98] and [2] new *upper eigenvalue bounds* for that twisted Dirac operator in terms of the L^2-norm of the mean curvature, results I obtained in my PhD thesis. Combining with the first statement, this yields upper bounds for the small eigenvalues of the Euler operator and we show on examples that they are sharp. This article and the corresponding chapter in my PhD thesis were the starting point for the construction of topological obstructions to Lagrangian embeddings [Hijazi, Montiel & Urbano '06].

4. **Remarques sur le spectre de l'opérateur de Dirac** [4]

   • Nicolas Ginoux, *Remarques sur le spectre de l'opérateur de Dirac*, C. R. Acad. Sci. Paris Sér. I **337** (2003), no. 1, 53–56

Abstract: We describe a new family of examples of hypersurfaces in the sphere satisfying the limiting-case in C. Bär's upper bound for the smallest eigenvalue of the Dirac operator.

Comments: If it is relatively elementary to show that the small Dirac eigenvalues of a compact oriented hypersurface in the round sphere can be bounded above by a Willmore-type functional (depending only on the L^2-norm of the mean curvature) [C. Bär '98], the question whether only round hyperspheres can satisfy the equality had remained unanswered. Basing on elementary calculations of eigenvalues on Riemannian products, we showed in this article that *all* so-called generalized Clifford tori (products of spheres "standardly" embedded in the sphere) fulfil that equality. This contrasts sharply with the situation of the scalar Laplacian, where only *minimal* hypersurfaces can satisfy the limiting case in Reilly's inequality.

3. **Une nouvelle estimation extrinsèque du spectre de l'opérateur de Dirac** [3]

   • Nicolas Ginoux, *Une nouvelle estimation extrinsèque du spectre de l'opérateur de Dirac*, C. R. Acad. Sci. Paris Sér. I **336** (2003), no. 10, 829–832

   Abstract: We prove a new upper bound for the smallest eigenvalues of the Dirac operator on a compact hypersurface of the hyperbolic space.

   Comments: Based on my PhD thesis, this article presents an *optimal upper bound* for the smallest Dirac eigenvalue in terms of the *L^∞-norm of the mean curvature* of a compact hypersurface of the hyperbolic space, establishing the analogous result to Ernst Heintze's for the first Laplace eigenvalue. In particular, it enhances the former estimate due to Christian Bär and that was never sharp.

2. **Reilly-type spinorial inequalities** [2] (shorter version in [P1])

   • Nicolas Ginoux, *Reilly-type spinorial inequalities*, Mathematische Zeitschrift **241** (2002), no. 3, 513–525

   Abstract: We give a new extrinsic upper bound for the smallest eigenvalues of the Dirac operator of a hypersurface. If the ambient manifold is the hyperbolic space, we show that its limiting case is achieved only for geodesic spheres.

   Comments: Given a closed immersed Riemannian hypersurface M in some model space, is there any *a priori* bound for the spectrum of the Dirac operator on M (when defined) in terms of extrinsic geometric quantities? In particular, does there exist any estimate involving the mean curvature only? Before I investigated it, this issue had been last considered by Christian Bär who had shown that, if the ambient manifold is a spaceform of non-negative curvature, then there are upper bounds *à la Reilly* (depending only on the averaged total squared mean curvature) for the small Dirac eigenvalues. His estimates were based on the min-max principle applied to the restriction of parallel or real Killing spinors onto the hypersurface. In the last case where the ambient manifold is the hyperbolic space, Christian Bär had obtained an estimate but which was never sharp, due to the fact that the corresponding Killing spinors do not have constant length. In the article, based on my PhD thesis, I overcame this technical difficulty by proving that, thanks to the conformal covariance of the Dirac operator, a *uniform* Reilly-type estimate can be proven for any M *conformally* immersed into some manifold with parallel or real Killing spinors, estimate which is moreover *sharp* for geodesic hyperspheres.

1. **On eigenvalue estimates for the submanifold Dirac operator** [1]

   • Nicolas Ginoux and Bertrand Morel, *On eigenvalue estimates for the submanifold Dirac operator*, Int. J. Math. **13** (2002), no. 5, 533–548

   Abstract: We give lower bounds for the eigenvalues of the submanifold Dirac operator in terms of intrinsic and extrinsic curvature expressions. We also show that the limiting cases give rise to a class of spinor fields generalizing that of Killing spinors. We conclude by translating these results in terms of intrinsic twisted Dirac operators.

   Comments: The main object of that article was to give, on any compact Riemannian spin manifold endowed with an arbitrary Riemannian spin bundle, sharp lower bounds *à la Friedrich* (in terms of weak curvature invariants) for operators of the form twisted Dirac operator plus potential. In particular, assuming *weak conditions about the potential and the curvature* of the twist bundle, we established several kinds of *geometric lower bounds* for the first positive eigenvalue of the above-mentioned operator. Those estimates apply in particular when the manifold is isometrically immersed into another Riemannian manifold. In that case, choosing the potential to be the norm of the mean curvature and the twist bundle to be the normal bundle, we obtained geometric lower eigenvalue bounds for an operator that is naturally attached to the immersion.

B2. **The Dirac spectrum** [B2]

• Nicolas Ginoux, *The Dirac spectrum*, Lecture Notes in Mathematics **1976** (2009), Springer

Summary: This book surveys the spectral properties of the spin Dirac operator. After a brief introduction to spin geometry, we present the main known estimates for Dirac eigenvalues on compact manifolds with or without boundary. We give examples where the spectrum can be made explicit and a chapter dealing with the non-compact setting. The methods mostly involve elementary analytical techniques and are therefore accessible for Master students entering the subject. We include a complete and updated list of references.

Comments: The spectral theory of Dirac-type operators has undergone spectacular progress in the last decades. In particular, questions of extrinsic geometry or metric rigidity could be very efficiently handled with spinorial techniques. None of the discoveries appeared after 2002 could be found in the last surveys by Christian Bär and Oussama Hijazi. The double goal of our enlarged survey article was to draw an up-to-date and coherent panorama of all known results and techniques related to the spectrum of the Dirac operator, and simultaneously to make it accessible to every student with a good differential-geometric background.

B1. **Wave equations on Lorentzian manifolds and quantization** [B1] (summary of the analytical part in [P2])

• Christian Bär, Nicolas Ginoux and Frank Pfäffle, *Wave equations on Lorentzian manifolds and quantization*, ESI Lectures in Mathematics and Physics (2007), EMS Publishing House

Summary: This book provides a detailed introduction to linear wave equations on Lorentzian manifolds (for vector-bundle valued fields). After a collection of preliminary material in the first chapter one finds in the second chapter the construction of local fundamental solutions together with their Hadamard expansion. The third chapter establishes the existence and uniqueness of global fundamental solutions on globally hyperbolic spacetimes and discusses Green's operators and well-posedness of the Cauchy problem. The last chapter is devoted to field quantization in the sense of algebraic quantum field theory. The necessary basics on C*-algebras and CCR-representations are developed in full detail.

Comments: The motivation for this work was two-fold: on the one hand it aimed at writing down in a *clear and self-contained way* the "well-known" *construction of global fundamental solutions* to linear wave operators on globally hyperbolic spacetimes, on the other hand at *setting up an adapted framework* for the mathematical foundations of particular *quantum field theories* based on wave equations. Both were and are considered as highly relevant by physicists.

P4. **Optimization of Tire Noise by Solving an Integer Linear Program (ILP)** [P4]

• Matthias Becker, Nicolas Ginoux, Sébastien Martin and Zsuzsanna Róka, *Optimization of Tire Noise by Solving an Integer Linear Program (ILP)*, in: 2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC 2016), October 9-12, 2016, Budapest.

Abstract: One important aim in tire industry when finalizing a tire design is the modelling of the noise characteristics as received by the passengers of the car. In previous works, the problem was studied using heuristic algorithms to minimize the noise by looking for a sequence under constraints. These constraints are imposed by tire industry. We present a new technique to compute the noise. We also propose an integer linear program based on that technique in order to solve this problem and find an optimal sequence. Our study shows that the integer linear programming approach shows significant improvement of the found tire designs, however it has to be improved further to meet the calculation time restrictions for real world problem size.

Comments: The noise produced by a (car-) tire can be modelled as a scalar multiple of the "largest" – in a sense to be made precise – Fourier coefficient of the periodic function giving the tire profile. That profile, in a simplified version where only one track is assumed to exist at the surface of the tire, can be described by a sequence of pitches of different types (in particular lengths) separated by groves. The number and length of each pitch-type is allowed to vary in some range, as is the total number of pitches at the surface of the tire. Since in practice the number of pitches makes the number of possible sequences much too high for all to be tested, algorithms are necessary to find an optimal solution within reasonable time. In this work, we could for the first time describe a method based on integer linear programming – which is already a surprise because of the deeply nonlinear nature of the problem – and allowing for the computation of a least approximative noise in a more efficient way than former algorithms.

H. **Analysis on Kähler and Lorentzian manifolds** [H]

• Nicolas Ginoux, *Analysis on Kähler and Lorentzian manifolds*, Habilitation thesis, Universität Regensburg, 2014

Summary: This habilitation thesis deals with the interactions between geometry of and analysis on smooth manifolds in different situations. Chapter 1 summarises all the results obtained in the next chapters. Chapter 2 deals with the spectrum of the Dirac operator of the Berger metrics on a 3-dimensional space. Estimates on the smallest eigenvalues of twisted Dirac operators on the complex projective space are computed and their limiting-case discussed in chapter 3. Chapter 4 focuses on a purely geometric question of classifying those Kähler spin manifolds with imaginary Kählerian Killing spinors. In chapter 5, we address the Yamabe problem on globally hyperbolic spacetimes. Chapter 6 is concerned with locally covariant quantization of fields for a large class of differential operators on spacetimes.

Comments: Several works are collected with apparently very different subjects but all dealing more or less with *analysis on manifolds*. Except the first one, all chapters are published papers.

D. **Opérateurs de Dirac sur les sous-variétés** [D]

• Nicolas Ginoux, *Opérateurs de Dirac sur les sous-variétés*, PhD thesis, Université Henri Poincaré - Nancy 1, 2002

Summary: In this PhD thesis, we study the spectrum of two Dirac operators defined on a submanifold. First, we prove a lower bound for an operator which is canonically associated with the Dirac-Witten operator. We then show that equality holds in these inequalities only if the submanifold admits a "twisted Killing" spinor. On the other hand, we give extrinsic upper bounds for the smallest eigenvalues of the Dirac operator on the submanifold twisted with its normal bundle. Completing C. Bär's work for hypersurfaces of the hyperbolic space, we obtain new estimates for hypersurfaces of manifolds admitting twistor-spinors. We finally extend these results to submanifolds of some particular Kählerian manifolds. The existence of Kählerian Killing spinors on such manifolds yields new eigenvalue estimates for CR-submanifolds. As a consequence, we obtain a comparison theorem for the eigenvalues of Dirac operators between Kählerian submanifolds of the complex projective space.

Comments: The original rough target for that PhD thesis consisted in relating Dirac eigenvalues with geometric and conformal invariants. As a starting point, I aimed at generalizing for the Dirac operator El Soufi and Ilias' inequality for the first positive Laplace eigenvalue in terms of a Willmore-type functional on a given compact submanifold of a real spaceform. In case the ambient spaceform is the real hyperbolic space and the codimension is 1, I could rapidly enhance the former unsharp upper bound by Christian Bär [C. Bär '98] and obtain a Heintze-type estimate; this result was actually published much later in [3], see comments above. Then I looked for a connection between the Dirac spectrum and the so-called *conformal volume*, see definition above. I could not find any – which was not bad luck, as I discovered much later [11] – but could combine the min-max principle with the conformal covariance of the Dirac operator in a clever way so as to obtain, in case a given compact manifold is e.g. conformally immersed as a hypersurface in the sphere, an "almost conformal" upper bound of the form Willmore functional plus a geometrically defined term for the small Dirac eigenvalues [2]. Pushing further the min-max technique, I obtained extrinsic upper eigenvalue estimates for Kähler or Lagrangian submanifolds of complex spaceforms [5], see also [13]. I also had the opportunity to collaborate with my "doctoral brother" Bertrand Morel and determine geometric lower bounds for operators of the form twisted Dirac operator plus potential [1].

M. **Géométrie hermitienne et géométrie spinorielle conforme** [M]

• Nicolas Ginoux, *Géométrie hermitienne et géométrie spinorielle conforme*, Master thesis, Université Henri Poincaré - Nancy 1, 1998

Summary: This master thesis consists of two independent parts. In the first one, basics of Hermitian geometry are reviewed in order to prove a result by Paul Gauduchon stating the existence and uniqueness of a Hermitian connection of minimal torsion on a given almost-Hermitian manifold. In the second one, we study spinors lying in the kernel of a Penrose-type operator on conformal spin manifolds according to Volker Buchholz's master thesis.

Comments: In the second part of that master thesis on Weyl structures, I discovered a minor gap in Volker Buchholz's master thesis. This led him to complete his results in a later publication [V. Buchholz, JGP '00].