Despite several well-known books and survey papers devoted to applications of Carleman estimates to various inverse problems, such as \cite{Lavrent'ev1980,Isakov1998,Klibanov2004,Bukhgejm1988}, this small booklet has a special interest by its simplicity and clarity in explosion of the techniques. Based on Carleman inequalities, the author provides a unified approach to the study of both elliptic Cauchy problems and some elliptic inverse problems. The author presents a simple way to obtain Carleman inequalities; namely, it is based on integration by parts. The first kind of such inequalities are applied to deriving three-ball inequalities for solutions of elliptic operators which is important for proving stability estimates for Cauchy problems. The second kind of these is used to establish the local behaviour of harmonic functions at the interior of the domain as well as at the boundary. These local estimates are the main tool for obtaining stability estimates for some classical inverse problems for elliptic equations with a single measurement. The booklet contains four chapters with twelve sections. In the first chapter, some basic notations on function spaces and assumptions on the geometry of the domain considered in the book are described. The second chapter is devoted to the uniqueness of continuation and Cauchy problems. The author obtained a Carleman inequality in the two-dimensional case and applied it to getting a stability estimate for the Cauchy problem and deduced from it a stability estimate for an inverse boundary coefficient problem. Next, the author obtained a Carleman inequality for a family of operators and applied it to get three-ball inequalities and the uniqueness of continuation for elliptic equations as well as stability estimates for the Cauchy problem. Chapter 3 “Determining the surface impedance of an obstacle from the scattering amplitude” is devoted to establish a stability estimate for the problem of recovering the surface impedance of an obstacle from the far field pattern. The main tool for the analysis if a Carleman inequality with constants depending explicitly on some bounds on the weight function. The last section “Determining a corrosion coefficient from a boundary measurement and an attenuation coefficient from an internal measurement” is devoted to the problem of determining the corrosion coefficient from a single electric measurement and to the problem of determining an attenuation coefficient from an energy density.

This book is well written and I highly recommend it to those who are interested in inverse problems for partial differential equations.

Reviewer: Dinh Nho Hào (Hanoi)

MSC:
- 35R30 Inverse problems for PDE
- 35-02 Research monographs (partial differential equations)
- 35Q92 PDEs in connection with biology and other natural sciences
- 35J60 Nonlinear elliptic equations

Keywords:
- Carleman inequalities; Cauchy problem; inverse problems

Full Text: DOI